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Light Does Not Tire

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It is shown that a true "tired light" effect cannot occur in any Lorentz- and gauge-invariant electromagnetic theory that reduces to Maxwell's theory for weak fields.

A general expansion of the universe has been the most widely accepted interpretation of galactic red shifts since the early work of Hubble. There have been many attempts, however, to attribute part or all of these red shifts to some sort of progressive loss of energy by radiation as it traverses very great distances. Gravitational drag on light (e.g., Zwicky, 1929) and photon decay processes (e.g., Jaakkola et al., 1975) have been among proposals of this type. Such ideas are often referred to as "tired light" theories; but that name is rather misleading, since it suggests that free radiation, uncoupled to anything else, would undergo a gradual loss of energy. The purpose of this note is to show that no true "tired light" theory is viable. Theories involving a gravitational or other interaction of light with matter are not affected by this proof, but, in view of the continuing uncertainties in cosmology concerning quasar red shifts and other important topics, it seems worthwhile to close at least one door. In addition, the simple result that will be obtained concerning nonlinear field equations is of interest in itself.

Our main result is the following theorem: all Lorentz- and gauge-invariant systems of vacuum electromagnetic field equations that reduce to Maxwell's equations for weak fields admit the usual plane-wave solutions of Maxwell's theory. The proof is straightforward. The Lagrangian density for such a theory can be written as a function of the two invariants $I = \mathbf{E}^2 - \mathbf{B}^2$ and $J = \mathbf{E}\cdot\mathbf{B}$. The Lagrangian density for Maxwell's theory is

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proportional to I, and that for any theory that reduces to Maxwell's for weak fields can be written as a Taylor expansion,

$$\mathscr{L} = \sum_{n,m=0}^{\infty} C_{nm} I^n J^m$$

The Born–Infeld (1934) electrodynamics or quantum electrodynamics below the pair-production threshold (Weisskopf, 1936) can, for example, be described in this way. The term $C_{01}J$ gives a vanishing contribution to the field equations, and all other terms containing odd powers of J would violate conservation of parity.

The field equations for this general nonlinear electrodynamics need not be written down in detail, for it now suffices to note that null fields with $I = 0$ and $J = 0$ will be solutions of the general field equations as long as Maxwell's equations are satisfied. (All terms in the field equations beyond those that arise from terms in $\mathscr{L}$ that are linear in I and J will contain a factor of I, J, or a first derivative of one of these invariants.) Thus the plane-wave solutions of Maxwell's theory are also solutions of the general nonlinear field equations. The same is true of any superposition of plane waves traveling in a given direction. Eddington (1923) pointed out a special case of this result long ago.

This special class of solutions of nonlinear field equations and the limited possibility of superposition of such solutions is itself notable. The conclusion of cosmological interest is that light waves described by a very general nonlinear electrodynamics would propagate without change of wavelength or frequency in a static, Euclidean space. This does not rule out the possibility that nonlinear interactions between light and, for example, static electric or magnetic fields might give rise to some red shift. However, light by itself would show no such effect.

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